Appendix A: The Hourly Auction Problem

A1 Definitions

Indices:

z: index denoting aggregate busses (geographic and foreign country zones). $z = 1,2,...,N$ where N is defined below.

i, j: indices used to denote aggregate busses between which there exists a power exchange interface. $i,j \in \{1,2,...,N\}$.

α : index used to denote some (real, aggregate, or virtual) intra-zone transmission line over which real power flow is limited for various reasons, such as a thermal real power transmission limitation or a stability induced limitation. $\alpha = 1,2,...,M$.

N: The total number of aggregate busses modeled. In the enhanced production grade model, the value of N is not expected to deviate significantly from 20.

M: The total number of (real, aggregate, or virtual) intra-zone transmission lines monitored against congestion. It is expected that the value of M will not deviate significantly from 10.

k_c: index denoting a consumption bid, $k_c=1,2,...K_C$. It is expected that K_C will be originally small (of the order of N) and eventually grow to the order of thousands.

k_g: index denoting a generation offer $k_g=1,2,...K_G$. It is expected that K_G will not exceed the order of a few thousands.

Input Variables:

N, K_C, K_G: number of aggregate busses, consumption bids, and generation offers, including must run and bilateral contracts.

$C_{ji}=C_{ij}$: $\neq 0$ if aggregate bus i is connected with aggregate bus j, 0 otherwise. In the enhanced production grade model interconnection topology, it is expected that there will be no more than Nx(N-1) non zero C_{ij} values for $i \neq j$, and $i,j \in \{1,2,...,N\}$. C_{ij} values are given inputs.

PV_{k_g}, QOV_{k_g} : price-quantity (loss adjusted) pair associated with generation offer k_g for all $k_g=1,2,...K_G$. Multiple offers associated with the same offer price are ranked according to a priority assigned to each offer.

$QVMIN$: minimum generation quantity accepted if a generation offer is accepted at all.

PA_{k_c}, QOA_{k_c} price-quantity (loss adjusted) pair associated with consumption bid k_c, for all $k_c=1,2,...K_C$. Multiple bids associated with the same bid price are ranked according to a priority assigned to each bid.

$MAXF_{ij}$: Maximum flow allowed over the interconnection from aggregate buss i to aggregate bus j, for all $i \neq j$, and $i,j \in \{1,2,...,N\}$ such that $C_{ij} \neq 0$.

S_{ij}^{z}: Contribution of one MW of net injection into aggregate bus z to the real power flow over the inter-zone power exchange interface connecting zone i to zone j. These are calculated from the C_{ij} coefficient inputs reflecting the appropriate impedance values when loops are present.

A_{α}^{z}: Contribution of one MW of net injection into aggregate bus z to the real power flow over some (real, aggregate, or virtual) intra-zone transmission line α. This input variable is expected to be provided for all $z = 1,2,...,N$ and $\alpha = 1,2,...,M$. Since this transmission line is internal to an aggregate bus/zone, it is not explicitly modeled as a power exchange interface. This (real, aggregate, or virtual) intra-zone transmission line is included in the power system model coupling various generators in the same constraint. This constraint is associated with the fact that real power flow over the (real, aggregate, or virtual) transmission line α is limited for various reasons such as a thermal real power transmission limitation or a stability induced limitation.

b_{α}: Maximum value of allowable power flow over (real, aggregate, or virtual) transmission line α for all $\alpha = 1,2,...,M$.

Output Variables:

QV_{k_g}, QA_{k_c} loss adjusted accepted generation offer and consumption bid quantities for all k_g, k_c.

ρ_z: Day Ahead Market clearing price in aggregate bus z.

λ: Dual variable value (Lagrange multiplier) associated with the energy balance constraint 4.

μ_{ij} for all for all $i \neq j$, and $i,j \in \{1,2,...,N\}$ such that $C_{ij} \neq 0$: Dual variable values (Lagrange multipliers) associated with the inter-zone power exchange interface constraints 5 of section A2. Note that μ_{ij} will be always zero when inter-zone power exchange interface from aggregate busses i to aggregate bus j is not binding. For a similar reason, at least one of μ_{ij} or μ_{ij} will be always zero.

ν_{α} for all $\alpha = 1,2,...,M$: Dual variable values (Lagrange multipliers) associated with the intra-zone transmission constraints 6 of section A2. Note that if the associated constraint 6 is not binding, then $\nu_{\alpha} = 0$.

$\rho_z = \lambda - \sum_{ij\, s.t.\, C_{ij}=1} \mu_{ij} \cdot S_{ij}^{z} - \sum_{\alpha=1,2,...,M} \nu_{\alpha} \cdot A_{\alpha}^{z}$ for all $z = 1,2,...,N$: Market clearing price in aggregate bus (or zone) z.

$\rho_{SenzaVincoliScambio}$: National Day Ahead Market clearing price in the absence of inter-zone power exchange or intra-zone transmission constraints. This is obtained by solving the mathematical problem in A2 in the absence of congestion constraints.

Slack on inter zone power exchange interfaces for use in the reserve market auction.

A2 Mathematical Optimization Problem Employed to Determine Offer/Bid Acceptance and Clearing Prices

Objective Function: Maximize Consumer plus Producer Surplus

1) $$\underset{QA_{k_c},QV_{k_g}}{Max}\left\{\sum_{k_c=1}^{K_C} PA_{k_c}\cdot QA_{k_c} - \sum_{k_g=1}^{K_G} PV_{k_g}\cdot QV_{k_g}\right\}$$

Subject to constraints:

2) $0 \le QA_{k_c} \le QOA_{k_c}$ for all $k_c \in \{1,2,...,K_C\}$

Continuous consumption bid quantity constraint.

3) $0 \le QV_{k_g} \le QOV_{k_g}$ for all $k_g \in \{1,2,...,K_G\}$

Generation offer capacity constraint without a minimum acceptable quantity.

4) $$\sum_{k_c=1}^{K_C} QA_{k_c} = \sum_{k_g=1}^{K_G} QV_{k_g}$$ Energy balance constraint.

5) $$\sum_{z=1}^{N} S_{ij}^{z}\left[\sum_{k_g\in \text{ aggr bus } z} QV_{k_g} - \sum_{k_c\in \text{ aggr bus } z} QA_{k_c}\right] \le MAXF_{i,j}$$ for all $i,j \in \{1,2,...,N\}$ s. t. $C_{ij} \neq 0$, $i \neq j$

6) $$\sum_{z=1}^{N} A_{\alpha}^{z}\left[\sum_{k_g\in \text{ aggr bus } z} QV_{k_g} - \sum_{k_c\in \text{ aggr bus } z} QA_{k_c}\right] \le b_{\alpha}$$ for all $\alpha = 1,2,...,M$.

Congestion monitoring of all potentially binding (real, aggregate, or virtual) intra-zone transmission constraints.

A3 Solution Technique

Step 1: The mathematical optimization problem described in section A2 is solved as a linear program (LP) by using all six constraints.

Step 2: The mathematical optimization problem described under section A2 above is next solved in the absence of constraints 5 and 6. The resulting energy balance constraint dual variable (Lagrange multiplier) value is the Day Ahead Market clearing price in the absence of inter zone power exchange and intra zone transmission constraints. This provides the value of $\rho_{\text{SenzaVincoliScambio.}}$.

Handling of Degenerate Solutions

Both in step 1 and in step 2, solution degeneracy of the type shown in figure 1 is taken care of by maximizing consumption sustainable at the same clearing price.

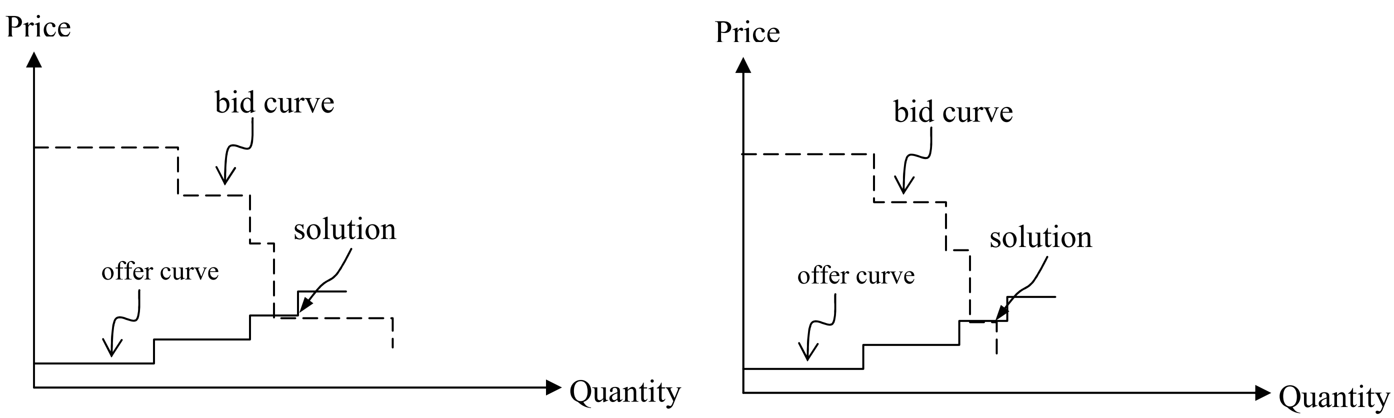


Figure 1: Degenerate Solution Handling. In cases where degenerate (multiple) solutions exist, the unique solution selected in an optimization post processor is the one that maximizes accepted bids for the clearing price subject to power exchange and other intra-zonal transmission constraints.

In the unlikely event that the supply and demand curves intersect at a vertical portion of both curves, the generation cost will determine the clearing price as shown in figure 2 below. To resolve this degeneracy, the clearing price of a zone is obtained from the reduced cost coefficient in the final LP tableau corresponding to an artificial offer with zero capacity and price that is introduced for this purpose. The resolution of degeneracy as described above is achievable when the price of additional supply exceeds the price of the last MWh of bid accepted. If the price of additional supply is smaller than the price of the last MWh of bid accepted, it is possible that numerical round off errors may result in making the next offer price the clearing price (see figure 3). Priority determines which offers and bids are accepted when not all of the bids/offers associated with the same price are accepted. This is achieved by appropriate modifications of bid and offer prices by an amount less than one cent of a Euro. This achieves the desired priority implementation without distorting prices that are reported to the closest cent of a Euro.

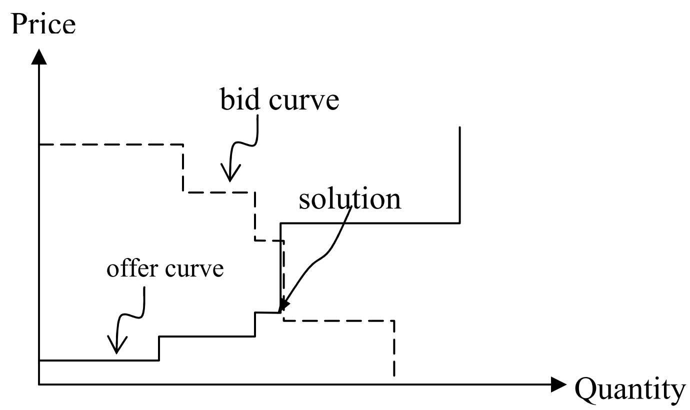


Figure 2. Degenerate solution handling in the unlikely event that total demand and total supply form a crossing vertical step at the same quantity value. Case one where the price of additional offer is larger than the price of the last MWh of bid accepted.

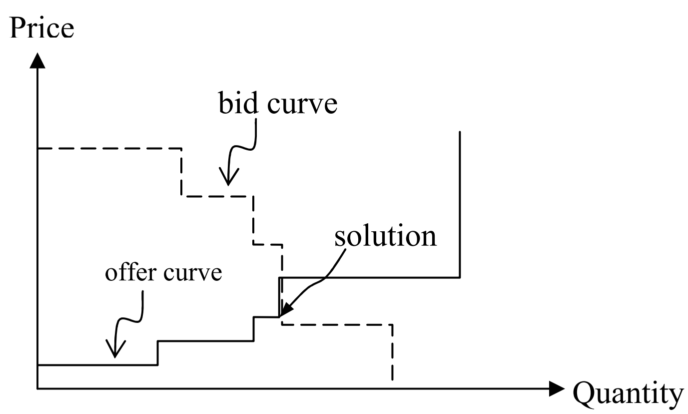
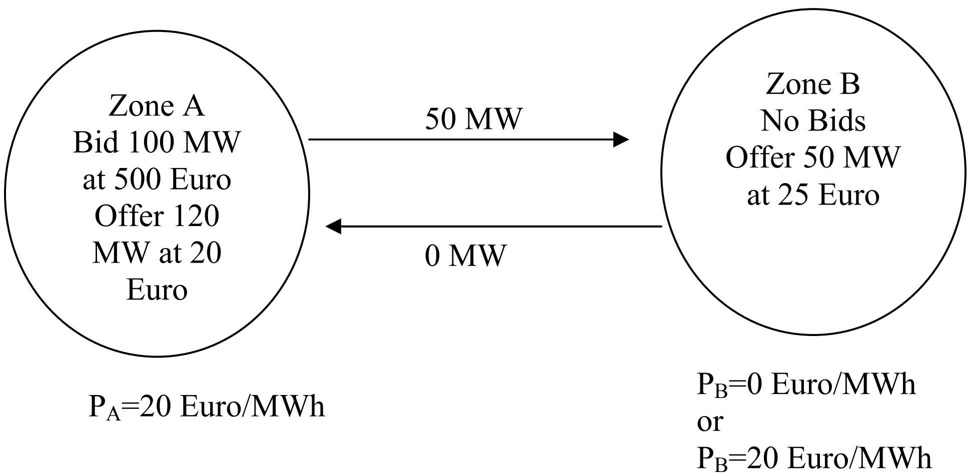


Figure 3. Degenerate solution handling in the unlikely event that total demand and total supply form a crossing vertical step at the same quantity value. Case two where the price of additional offer is smaller than the price of the last MWh of bid accepted.

Appendix B: Ambiguities in the Determination of Marginal Cost Prices in Zones with No Bids and Zero Export Interconnection Capacity

In early testing, TCA observed that when export interconnection capacity was set to zero in a zone with no bids, the standard market clearing algorithm returned a zonal price equal to 0. This price is indeed the correct opportunity cost of incremental generation in that zone. However, the marginal cost of incremental demand might be larger than 0. In the example below, due to the zero export constraint, the incremental opportunity cost of generation in zone B is 0 Euro/MWh while the cost of incremental demand is 20 Euro/MWh, the zonal price in Zone A which would satisfy an incremental bid in zone B.



The ambiguity of the marginal cost price in Zone B is caused by the discontinuity of the derivative of cost with respect to net injection in zone B, otherwise known as discrepancy between left and right derivatives. When a phenomenon of this sort occurs, we observe that the cost derivative does not exist. Instead, what exists is a sub-gradient bounded by the left and right derivative. The existence of a sub-gradient in the example above prescribes that $0 \le P_B \le 20$ Euro/MWh , namely that the acceptable price lies inside a range of acceptable prices. In all cases, except for situations with zones featuring zero bids and no allowed exports, the left and right derivatives coincide and the range of prices observed in the above example collapses to a single, unique point.

The TCA algorithm estimates zonal prices as the marginal opportunity cost of incremental generation, namely it uses the derivative of costs with respect to positive net injections (the derivative from the right). In an early version of the market-clearing algorithm, a heuristic rule was implemented that set the zonal price equal to the incremental cost of demand. It was thought at the time that this was preferable from an "aesthetic" point of view. Since zonal prices are irrelevant in the absence of bids and allowable exports, this adjustment indeed had an aesthetic rather than a practical motivation. With the extended use of the standard algorithm in the Adjustment markets, however, it was realized that situations where one or more zones featured zero bids and no exports were allowed, were quite common. In some of these cases, the heuristic rule gave occasionally incorrect results by erroneously estimating the left derivative as the cost of incremental imports without accounting for zonal offer incremental costs. To avoid what would have been a time consuming algorithmic implementation that would have been required to always determine accurately left and right derivatives, we opted to remove the heuristic rule altogether, and report always the correct right derivative. We therefore report the marginal opportunity cost of incremental generation (i.e. positive injection) as opposed to the sometimes different incremental marginal cost of demand. In conclusion, whenever a range of zonal prices is applicable, the algorithm reports the lower bound of that range.

